On rps-separation axioms

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ABSTRACT

The authors introduced rps-closed sets and rps-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The aim of this paper is to introduce rps-$T_{½}$, rps-$T_{⅓}$, rps-T_b, rps-$T_{¾}$ spaces and characterize their basic properties.

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1. INTRODUCTION

Separation axioms in topological spaces play a dominated role in analysis. Recently general topologists concentrate on separation axioms between T_0 and T_1. In this paper the concepts of rps-$T_{½}$, rps-$T_{⅓}$, rps-T_b, rps-$T_{¾}$ spaces are introduced, characterized and studied their relationships with $T_{½}$ space[7], semi-$T_{½}$ space[3], pre-regular-$T_{½}$ space[5], semi-pre-$T_{½}$ space[4], pgpr-$T_{½}$ space[2], pre-semi-T_b space[14], pre-semi-$T_{½}$ space[14], pre-semi-$T_{¾}$ space[14], that are respectively introduced by Levine, Bhattacharya, Gnanambal, Dontchev, Anitha, Veerakumar and their collaborators.

2. PRELIMINARIES

Throughout this paper (X,τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X \ A denotes the complement of A in X. Throughout the paper □ indicates the end of the proof. We recall the following definitions and results.

Definition 2.1

A subset A of a space (X,τ) is called

(i) regular-open [13] if A = *int cl*A and regular-closed if A = *cl int*A.

(ii) semi-open [6] if A $\subseteq$ *cl int*A and semi-closed if *int cl*A $\subseteq$ A.

(iii) pre-open [9] if A $\subseteq$ *int cl*A and pre-closed if *cl int*A $\subseteq$ A.

(iv) semi-pre-open [1] if A $\subseteq$ *cl int cl*A and semi-pre-closed if *int cl int*A $\subseteq$ A.

Definition 2.2

A subset A of a space (X,τ) is called g-closed[7] (resp. rg-closed[10], resp. gsp-closed[4], resp. gpr-closed[5], resp. gp-closed[8], resp. pre-semiclosed[14], resp. pgpr-closed[2], resp. rps-closed[11], resp. sg-closed[3]) if *cl*A $\subseteq$ U (resp. *cl*A $\subseteq$ U, resp. *spcl*A$\subseteq$U, resp. *pcl*A $\subseteq$U, resp. *pcl*A $\subseteq$U, resp. *spcl*A$\subseteq$U, resp. *pcl*A $\subseteq$U, resp. *spcl*A$\subseteq$U, resp. *scl*A$\subseteq$U) whenever A$\subseteq$U and U is open (resp. regular-open, resp. open, resp. regular-open, resp. open, resp. g-open, resp. rg-open, resp. rg-open, resp. semi-open).

A subset B of a space X is called g-open if X \ B is g-closed. The concepts of rg-open, gsp-open, gpr-open, gp-open, pre-semiopen, pgpr-open, rps-open and sg-open sets can be analogously defined.

Definition 2.3

A space (X,τ) is called a $T_{½}$ space[7] (resp. $T^*_{½}$ space[10], resp. semi-$T_{½}$ space[3], resp. pre-regular-$T_{½}$ space[5], resp. semi-pre-$T_{½}$ space[4], resp. pre-semi-$T_{½}$ space[14], resp. pgpr-$T_{½}$ space[2], resp. pre-semi-T_b space[14], resp. pre-semi-$T_{¾}$ space[14], resp. gpr-$T_{½}$ space[2]) if every g-closed (resp. rg-closed, resp. sg-closed, resp. gpr-closed, resp. gsp-closed, resp. pre-semiclosed, resp. pgpr-closed, resp. pre-semiclosed, resp. pre-semiclosed, resp. gpr-closed) set is closed(resp. closed, resp. semi-closed, resp. pre-closed, resp. semi-pre-closed, resp. semi-pre-closed, resp. pre-closed, resp. semi-closed, resp. pre-closed, resp. pgpr-closed.

Lemma 2.4 [11] If a set A is rps-closed then *spcl*A \ A does not contain a non empty rg-closed set.

Definition 2.5 [15] A space X is called extremally disconnected if the closure of each open subset of X is open.

Lemma 2.6[5] Every pre-regular-$T_{½}$ space is semi-pre $T_{½}$.

Definition 2.7
A function f: X → Y is called

(i) semi-continuous [9] if $f^{-1}(V)$ is semi-closed in X for every closed set V in Y.

(ii) pre-continuous [9] if $f^{-1}(V)$ is pre-closed in X for every closed set V in Y.

(iii) semi-pre-continuous [1] if $f^{-1}(V)$ is semi-preclosed in X for every closed set V in Y.

(iv) pre-semicontinuous [14] if $f^{-1}(V)$ is pre-semiclosed in X for every closed set V in Y.

(v) rps-continuous [12] if $f^{-1}(V)$ is rps-closed in X for every closed set V in Y.

Lemma 2.8[11]

(i) Every rps-closed set is pre-semiclosed

(ii) Every pgpr-closed set is rps-closed.

(iii) Every semi-pre-closed set is rps-closed.

(iv) Every rps-closed set is gsp-closed.

Lemma 2.9 [2] In an extremally disconnected space, *pcl*A = *spcl*A.

Lemma 2.10[2] Every pre-closed set is pgpr-closed.

We use the following notations.

RPSO(X, τ) - The collection of all rps-open sets in (X, τ).

SPO(X, τ) - The collection of all semi-pre-open sets in (X, τ).

SPC(X, τ) - The collection of all semi-pre-closed sets in (X, τ).

RPSC(X, τ) - The collection of all rps-closed sets in (X, τ).

3. rps-T_k spaces where $k \in \{b, ½, ⅓, ¾\}$

As application of regular pre-semiclosed sets, four spaces namely, regular pre-semi-$T_{½}$ spaces, regular pre-semi-$T_{⅓}$ spaces, regular pre-semi-T_b spaces and regular pre-semi-$T_{¾}$ spaces are introduced. The following implication diagram will be useful in this paper.

Diagram 3.1

$$\begin{tikzcd}
\textit{semi-closed} \arrow[r] & \textit{semi-pre-closed} \arrow[r] & \textit{rps-closed} \arrow[r] & \textit{pre-semiclosed} \\
& \textit{pre-closed} \arrow[u] & &
\end{tikzcd}$$

Examples can be constructed to show that the reverse implications are not true. This motivates us to introduce the following spaces.

Definition 3.2

A space (X,τ) is called regular pre-semi-$T_{½}$(briefly rps-$T_{½}$) if every rps-closed set is semi-pre-closed.

Definition 3.3

A space (X,τ) is called regular pre-semi-$T_{⅓}$ (briefly rps-$T_{⅓}$) if every pre-semi-closed set is rps-closed.

Definition 3.4

A space (X,τ) is called regular pre-semi-T_b (briefly rps-T_b) if every rps-closed set is semi-closed.

Definition 3.5

A space (X,τ) is called regular pre-semi-$T_{¾}$ (briefly rps-$T_{¾}$) if every rps-closed set is pre-closed.

Theorem 3.6

(i) Every pre-semi-$T_{½}$ space is an rps-$T_{½}$ space.
(ii) Every semi-pre-$T_{½}$ space is an rps-$T_{½}$ space.
(iii) Every pre-regular-$T_{½}$ space is an rps-$T_{½}$ space.
(iv) Every rps-T_b space is an rps-$T_{½}$ space.

Proof:

Suppose X is pre-semi-$T_{½}$. Let V be an rps-closed set in X. Using Lemma 2.8(i), V is pre-semiclosed. Since X is pre-semi-$T_{½}$, using Definition 2.3, V is semi-pre-closed. This proves (i).
Suppose X is semi-pre-$T_{½}$. Let V be an rps-closed set in X. Using Lemma 2.8(iv), V is gsp-closed. Since X is semi-pre-$T_{½}$, using Definition 2.3, V is semi-pre-closed. This proves (ii).
(iii) follows from (ii) and Lemma 2.6.
(iv) follows from the fact that every semi-closed set is semi-pre-closed.

□

The converses of Theorem 3.6 are not true as shown in Example 4.1 and Example 4.2.

Theorem 3.7

(i) Every rps-$T_{¾}$ space is an rps-$T_{½}$ space.
(ii) Every rps-$T_{¾}$ space is a pgpr-$T_{½}$ space.
(iii) Every pre-semi-$T_{¾}$ space is an rps-$T_{⅓}$ space.

Proof

(i) follows from the fact that every pre-closed set is semi-pre-closed.
Suppose X is rps-$T_{¾}$. Let V be a pgpr-closed set in X. Using Lemma 2.8(ii), V is rps-closed. Since X is rps-$T_{¾}$, V is pre-closed. This proves (ii).
Suppose X is pre-semi-$T_{¾}$. Let V be a pre-semiclosed set in X. Since X is pre-semi-$T_{¾}$, using Definition 2.3, V is pre-closed. Using Lemma 2.10 and Lemma 2.8(ii), V is rps-closed. This proves X is an rps-$T_{⅓}$ space.

□

The converses of Theorem 3.7 are not true as shown in Example 4.3.

Theorem 3.8

Every pre-semi-T_b space is an rps-T_b space.

Proof

Suppose X is pre-semi-T_b. Let V be an rps-closed set in X. Using Lemma 2.8(i), V is pre-semiclosed. Since X is pre-semi-T_b, V is semi-closed. This proves the theorem. □

The converse of Theorem 3.8 is not true as shown in Example 4.1.

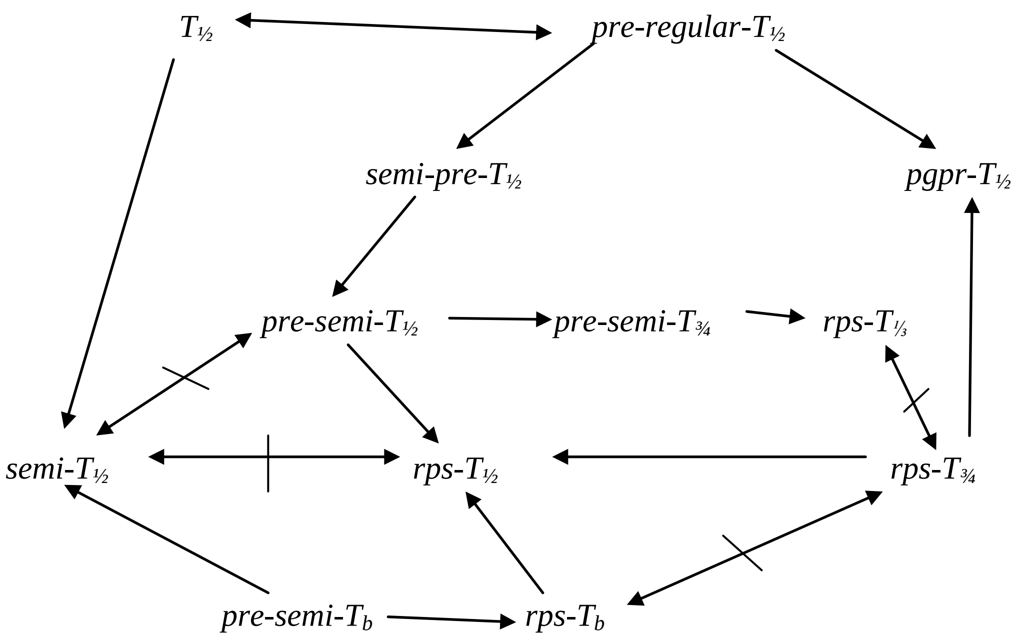
The concepts of rps-$T_{½}$ and semi-$T_{½}$ are independent as shown in Example 4.2 and Example 4.4.

The concepts of rps-T_b and rps-$T_{⅓}$ are independent with the concept of rps-$T_{¾}$ as shown in Example 4.1, Example 4.2 and Example 4.3.
From the above discussions and known results we have the following implication diagram. In this diagram by

A $\rightarrow$ B we mean A implies B but not conversely and

A $\not\leftrightarrow$ B means A and B are independent of each other.

Diagram 3.9



Theorem 3.10

A space X is rps-$T_{½}$ if and only if every singleton set is rg-closed or semi-pre-open.

Proof

Suppose X is rps-$T_{½}$. Fix $x \in X$. Suppose {x} is not rg-closed. Then X \ {x} is not rg-open. Then X is the only rg-open set containing X \ {x} and hence X \ {x} is trivially an rps-closed subset of (X,τ). Since X is rps-$T_{½}$, using Definition 3.2, X \ {x} is semi-pre-closed. Therefore {x} is semi-pre-open.

Conversely suppose every singleton set is rg-closed or semi-pre-open. Let A be rps-closed in X. Since A is rps-closed, by using Lemma 2.4, *spcl*A \ A does not contain a non empty rg-closed set. Let $x \in$ *spcl*A. By our assumption {x} is either rg-closed or semi-pre-open.

Case (i)

Suppose there is an element $x \in$ *spcl*A such that {x} is rg-closed. Since {x} is rg-closed, using Lemma 2.4, $x \notin$ *spcl*A \ A that implies $x \in A$. Therefore A = *spcl*A. Therefore A is semi-pre-closed.

Case (ii)

Suppose {x} is not rg-closed for all $x \in$ *spcl*A.

$x \in$ *spcl*A $\Rightarrow$ {x} is semi-pre-open

$\Rightarrow$ $\{x\} \cap A \neq \varnothing$

$\Rightarrow$ $x \in A$

$\Rightarrow$ A = *spcl*A

$\Rightarrow$ A is semi-pre-closed.

From Case (i) and Case (ii), it follows from Definition 3.2 that X is rps-$T_{½}$.

□

Theorem 3.11

Let X be an rps-$T_{⅓}$ space. Then

(i) X is rps-$T_{½}$ if and only if it is pre-semi-$T_{½}$.

(ii) X is rps-T_b if and only if it is pre-semi-T_b.

(iii) X is rps-$T_{¾}$ if and only if it is pre-semi-$T_{¾}$.

Proof

Suppose X is rps-$T_{½}$ and rps-$T_{⅓}$. Let A be pre-semiclosed in X. Using Definition 3.3, A is rps-closed. Since X is rps-$T_{½}$, by Definition 3.2, A is semi-pre-closed. Therefore X is pre-semi-$T_{½}$.

Conversely we assume that X is pre-semi-$T_{½}$. Suppose A is pre-semiclosed. Since X is pre-semi-$T_{½}$, using Definition 2.3, A is semi-pre-closed. Using Lemma 2.8(iii), A is rps-closed. This proves that X is rps-$T_{⅓}$.

Suppose B is rps-closed. Using Lemma 2.8(i), B is pre-semiclosed. Since X is pre-semi-$T_{½}$, using Definition 2.3, B is semi-pre-closed. Therefore X is rps-$T_{½}$. This proves (i).

Suppose X is rps-$T_{⅓}$ and rps-T_b. Let A be a pre-semi-closed set in X. Using

Definition 3.3, A is rps-closed. Since X is rps-T_b, by Definition 3.4, A is semi-closed. Therefore X is pre-semi-T_b.

Conversely we assume that X is pre-semi-T_b. Suppose A is a pre-semiclosed subset of X. Since X is pre-semi-T_b, A is semi-closed. Using Diagram 3.1, A is semi-pre-closed. Using Lemma 2.8(iii), A is rps-closed. This proves that X is rps-$T_{⅓}$. Suppose B is rps-closed. Using Lemma 2.8(i), B is pre-semiclosed. Since X is pre-semi-T_b, B is semi-closed so that X is rps-T_b. This proves (ii).

Suppose X is rps-$T_{⅓}$ and rps-$T_{¾}$. Let A be a pre-semiclosed set in X. Using

Definition 3.3, A is rps-closed. Since X is rps-$T_{¾}$, by using Definition 3.5, A is pre-closed. Therefore X is pre-semi-$T_{¾}$.

Conversely we assume that X is pre-semi-$T_{¾}$. Suppose A is pre-semiclosed. Since X is pre-semi-$T_{¾}$, A is pre-closed. Using Diagram 3.1, A is semi-pre-closed. Using

Lemma 2.8(iii), A is rps-closed. This proves that X is rps-$T_{⅓}$. Suppose B is rps-closed. Using Lemma 2.8(i), B is pre-semiclosed. Since X is pre-semi-$T_{¾}$, B is pre-closed so that X is rps-$T_{¾}$. This proves (iii).

□

Theorem 3.12

(i) If (X,τ) is an rps-$T_{⅓}$ space then for each $x \in X$, $\{x\}$ is either rg-closed or rps-open.
(ii) If (X,τ) is an rps-T_b space then for each $x \in X$, $\{x\}$ is either rg-closed or semi-open.
(iii) If (X,τ) is an rps-$T_{¾}$ space, then for each $x \in X$, $\{x\}$ is either rg-closed or pre-open.

proof

Suppose $\{x\}$ is not an rg-closed subset of an rps-$T_{⅓}$ space (X,τ). So $\{x\}$ is not g-closed. Then X is the only g-open set containing $X \setminus \{x\}$. Therefore $X \setminus \{x\}$ is pre-semiclosed since (X,τ) is rps-$T_{⅓}$, $X \setminus \{x\}$ is rps-closed or equivalently $\{x\}$ is rps-open. This proves (i).

Suppose $\{x\}$ is not an rg-closed subset of an rps-T_b space (X,τ). Then X is the only rg-open set containing $X \setminus \{x\}$ and hence $X \setminus \{x\}$ is rps-closed. Since (X,τ) is rps-T_b, $X \setminus \{x\}$ is semi-closed or equivalently $\{x\}$ is semi-open.

Suppose $\{x\}$ is not an rg-closed subset of an rps-$T_{¾}$ space (X,τ). Then $X \setminus \{x\}$ is not rg-open. X is the only rg-open set containing $X \setminus \{x\}$ and hence $X \setminus \{x\}$ is trivially an rps-closed subset of (X,τ). Since (X,τ) is rps-$T_{¾}$, $X \setminus \{x\}$ is pre-closed or equivalently $\{x\}$ is pre-open.

□

The converses of Theorem 3.12 are not true as shown in Example 4.2, Example 4.3 and Example 4.4.

Theorem 3.13

A space (X,τ) is rps-$T_{½}$ if and only if $SPC(X,\tau) = RPSC(X,\tau)$.
A space (X,τ) is rps-$T_{½}$ if and only if $SPO(X,\tau) = RPSO(X,\tau)$.

Proof

From Diagram 3.1, $SPC(X,\tau) \subseteq RPSC(X,\tau)$.

(X,τ) is rps-$T_{½}$ $\Rightarrow RPSC(X,\tau) \subseteq SPC(X,\tau)$

$\Rightarrow SPC(X,\tau) = RPSC(X,\tau)$.

Conversely suppose $SPC(X,\tau) = RPSC(X,\tau) \Rightarrow (X,\tau)$ is rps-$T_{½}$. This proves (i).

The result (ii) follows directly from result(i).

□

Theorem 3.14

Let X be an extremally disconnected space.
(i) If X is rps-$T_{½}$ and gpr-$T_{½}$, then it is pre-regular-$T_{½}$.
(ii) If X is $T^*_{½}$, then every gp-closed set is rps-closed.

Proof

Let X be an extremally disconnected space. Suppose X is rps-$T_{½}$ and gpr-$T_{½}$. Let A be gpr-closed in X. Since X is gpr-$T_{½}$, using Definition 2.3, A is pgpr-closed. Again using Lemma 2.8(ii), A is rps-closed. Since X is rps-$T_{½}$, using Definition 2.2, A is semi-pre-closed. Again using Definition 2.1(iv), $int\ cl\ int A \subseteq A$. Since X is extremally disconnected, it follows that $cl\ int A \subseteq A$. Therefore A is pre-closed that implies X is pre-regular-$T_{1/2}$.

This proves (i).

Suppose X is $T^{*}_{½}$, Let $A \subseteq U$, U be rg-open and A be gp-closed. Since X is $T^{*}_{½}$, by Definition 2.3, U is open. Since A is gp-closed, by Definition 2.2, $pclA \subseteq U$. Since X is extremally disconnected, using Lemma 2.9, $spclA = pclA \subseteq U$. Therefore A is rps-closed. This proves (ii).

□

Theorem 3.15

Let X be $T^{*}_{½}$ space. Then every gsp-closed set is rps-closed.

Proof

Suppose A is gsp-closed in X. Let $A \subseteq U$ and U be rg-open. Since X is $T^{*}_{½}$, by Definition 2.3, U is open and since A is gsp-closed, by Definition 2.2, $spclA \subseteq U$. Again using Definition 2.2, A is rps-closed.

□

Theorem 3.16

(i) If X is rps-$T_{½}$ then every rps-continuous function is semi-pre-continuous.
(ii) If X is rps-$T_{⅓}$ then every pre-semicontinuous function is rps-continuous.
(iii) If X is rps-T_b then every rps-continuous function is semi-continuous.
(iv) If X is rps-$T_{¾}$ then every rps-continuous function is pre-continuous.

Proof

Suppose X is rps-$T_{½}$. Let A be closed in Y and f: $X \rightarrow Y$ be rps-continuous. Since f is rps-continuous, using Definition 2.7(v), $f^{-1}(A)$ is rps-closed. Since X is rps-$T_{½}$, using Definition 3.2, $f^{-1}(A)$ is semi-pre-closed. This proves that f is semi-pre-continuous.

Suppose X is rps-$T_{⅓}$. Let A be closed in Y and f be pre-semicontinuous. Since f is pre-semicontinuous, using Definition 2.7(iv), $f^{-1}(A)$ is pre-semiclosed. Since X is rps-$T_{⅓}$, Definition 3.3, $f^{-1}(A)$ is rps-closed. This proves that f is rps-continuous.

Suppose X is rps-$T_{⅓}$. Let A be closed in Y and f be rps-continuous. Since f is rps-continuous, using Definition 2.7(v), $f^{-1}(A)$ is rps-closed in X. Since X is rps-T_b, using Definition 3.4, $f^{-1}(A)$ is semi-closed. Therefore f is semi-continuous.

Suppose X is rps-$T_{¾}$. Let A be closed in Y and f be rps-continuous. Since f is rps-continuous, using Definition 2.7(v), $f^{-1}(A)$ is rps-closed in X. Since X is rps-$T_{¾}$, using Definition 3.5, $f^{-1}(A)$ is pre-closed. Therefore f is pre-continuous.

□

Theorem 3.17

If X is pre-semi-$T_{½}$ and if f: $X \rightarrow Y$ then the following are equivalent.
(i) f is semi-pre-continuous.
(ii) f is pre-semicontinuous.
(iii) f is rps-continuous.

Proof

Suppose f is semi-pre-continuous. Let $A \subseteq Y$ be closed. Since f is semi-pre-continuous, $f^{-1}(A)$ is semi-pre-closed in X. Using Diagram 3.1, $f^{-1}(A)$ is pre-semiclosed. Therefore f is pre-semicontinuous. This proves (i) $\Rightarrow$ (ii).

Suppose f is pre-semicontinuos. Let $A \subseteq Y$ be closed. Since f is pre-semicontinuous, $f^{-1}(A)$ is pre-semiclosed. By Theorem 3.11(i), X is rps-$T_{⅓}$. Therefore $f^{-1}(A)$ is rps-closed. This proves (ii) $\Rightarrow$ (iii).

Suppose f is rps-continuous. Let $A \subseteq Y$ be closed. Since f is rps-continuous, $f^{-1}(A)$ is rps-closed. By Theorem 3.11(i), X is rps-$T_{½}$. Therefore $f^{-1}(A)$ is semi-pre-closed. This proves (iii) $\Rightarrow$ (i).

□

4. Examples

Example 4.1

Let X = {a,b,c} with τ = {Ø,X,{a},{a,c}}.Then (X,τ) is rps-$T_{½}$, rps-T_b and rps-$T_{¾}$ but not pre-semi-$T_{½}$, not semi-pre-$T_{½}$, not pre-regular-$T_{½}$, not pre-semi-T_b and not rps-$T_{⅓}$.

Example 4.2

Let X = {a,b,c} with τ = {Ø,{a,b},X}. Then (X,τ) is rps-$T_{½}$ and rps-$T_{¾}$ but not rps-T_b and not semi-$T_{½}$.

Example 4.3

Let X = {a,b,c} and τ = {Ø,{a},{b},{a,b},X}. Then (X,τ) is rps-$T_{⅓}$, rps-T_b, rps-$T_{½}$ and pgpr-$T_{½}$ but not pre-semi-$T_{¾}$ and not rps-$T_{¾}$.

Example 4.4

Let X = {a,b,c,d} with τ = {Ø, {a},{b},{a,b},{b,c},{a,b,c},X}. Then (X,τ) is semi-$T_{½}$ but not rps-$T_{½}$ and not rps-$T_{⅓}$.

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